Practice Test Derivatives

Name Answer Key

For problems 1 – 8, find dy/dx. Make sure you simplify problem #2 and #3

1) $y = \frac{1}{2}x^{10} - \frac{1}{6}x^6 + \frac{x}{3}$

$y = \frac{1}{2}x^{10} - \frac{1}{6}x^6 + \frac{1}{3}x$

$\boxed{y' = 5x^9 - x^5 + \frac{1}{3}}$

2) $\mathbf{y = (4x - 2)(7 - 4x^5)}$

Product Rule First

$y' = (4x-2)(-20x^4) + (7-4x^5)(4)$

$y' = -80x^5 + 40x^4 + 28 - 16x^5$

$\boxed{y' = -96x^5 + 40x^4 + 28}$

Alg First

$y = 28x - 16x^6 - 14 + 8x^5$

$\boxed{y' = 28 - 96x^5 + 40x^4}$

3) $\mathbf{y = \frac{2x + 6}{8x - 1}}$

$y' = \frac{(8x-1)(2) - (2x+6)(8)}{(8x-1)^2}$

$16x - 2 - 16x - 48$

$\boxed{y' = \frac{-50}{(8x-1)^2}}$

4) $y = \frac{10}{x^5} + \frac{11}{x} + 5x$

$y = 10x^{-5} + 11x^{-1} + 5x$

$\boxed{y' = -50x^{-6} - 11x^{-2} + 5}$

5) $y = (x^6)(\cot(\sec x))$

Product Rule

$\boxed{y' = x^6 \cdot (-\csc^2(\sec x) \cdot \sec x \tan x) + \cot(\sec x)(6x^5)}$

Quotient Rule

6) $y = \dfrac{\tan(5x)}{3x}$

$$y' = \frac{3x(\sec^2(5x)\cdot 5 - \tan(5x)(3)}{(3x)^2} = \frac{15x\sec^2(5x) - 3\tan(5x)}{9x^2}$$

Product Rule

7) $y = \csc(\ln x)\sec(e^x)$

$$y' = \csc(\ln x)\cdot\left(\sec(e^x)\tan(e^x)\cdot e^x\right) + \sec(e^x)\cdot\left(-\csc(\ln x)\cot(\ln x)\right)\cdot\frac{1}{x}$$

Product Rule

8) $y = (x^{\frac{4}{3}})(\cot(x))$ $\quad y = x^{4/3}\cot x$

$$y' = x^{4/3}(-\csc^2 x) + \cot x\left(\tfrac{4}{3}x^{1/3}\right)$$

Find the equation for the tangent to the curve at the given point.

9) $f(x) = \dfrac{5}{x+4}$ at $x = -1$

Point $f(-1) = \dfrac{5}{-1+4} = \dfrac{5}{3}$

$\left(-1, \frac{5}{3}\right)$

$$\boxed{y = \frac{5}{3} - \frac{5}{9}(x+1)}$$

Slope = derivative

$f(x) = \dfrac{5}{(x+4)}$

$f'(x) = \dfrac{(x+4)(0) - 5(\cancel{1})}{(x+4)^2}$

$f'(x) = \dfrac{-5}{(x+4)^2}$ $\quad f'(-1) = \dfrac{-5}{(-1+4)^2} = \dfrac{-5}{9}$

Solve the problem.

10) Find the equation of the normal line to the curve $y = 4x + 4x^2$ at the point $(-2, 8)$.

$$\boxed{y = 8 + \frac{1}{12}(x+2)}$$

$$\frac{dy}{dx} = 4 + 8x$$

$$\left.\frac{dy}{dx}\right|_{x=-2} = 4 + 8(-2) = 4 - 16 = -12$$

11) Find the points where the graph of the function has horizontal tangents.

$f(x) = 6x^2 + 5x + 2$

$$f'(x) = 12x + 5$$

$$0 = 12x + 5$$

$$-5 \quad\quad -5$$

$$\frac{-5}{12} = \frac{12x}{12}$$

$$x = -\frac{5}{12} \quad y = 6\left(\frac{-5}{12}\right)^2 + 5\left(\frac{-5}{12}\right) + 2$$

$$\boxed{\left(-\frac{5}{12}, \frac{23}{24}\right)}$$

12) **Find dy/dx if $y = 5\csc^7(2x^3)$ and then simplify the derivative**

1) PWR RULE
2) Derivative of Base
3) Derivative of Angle

$$y = 5(\csc(2x^3))^7$$

$$y' = 35(\csc(2x^3))^6 \cdot (-\csc(2x^3)\cot(2x^3)) \cdot 6x^2$$

$$\boxed{y' = -210x^2 (\csc(2x^3))^7 \cot(2x^3)}$$

13) **Find dy/dx if** $y = \sqrt{2x + e^{x^4}}$

$$y = (2x + e^{x^4})^{1/2}$$

$$y' = \frac{1}{2}(2x + e^{x^4})^{-1/2} \cdot (2 + e^{x^4} \cdot 4x^3)$$

14) **Find dy/dx if** $y = \sin(2x)\tan(5x - 4)$

Product Rule

$$y' = \sin(2x)\sec^2(5x-4) \cdot 5 + \tan(5x-4)(\cos(2x) \cdot 2)$$

15) a. Find the derivative of f(g(x)) $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

b. Use the table below and your answer to part a to find the derivative of g(~~f(x)~~)) evaluated at x=4

x	f(x)	g(x)	f '(x)	g '(x)
3	1	4	6	7
4	3	3	2	-6

$f(g(x))$

$f'(g(4)) \cdot g'(4)$

$f'(3) \cdot (-6)$

$6 \cdot (-6)$

-36

Find the derivative of the given function.

16) **y =9 arcsin(3x^5)**

$$y' = \frac{9}{\sqrt{1-(3x^5)^2}} \cdot 15x^4 = \boxed{\frac{135x^4}{\sqrt{1-9x^{10}}}}$$

17) **y = 10arccos(3t)**

$$y' = \frac{-10}{\sqrt{1-(3t)^2}} \cdot (3) = \boxed{\frac{-30}{\sqrt{1-9t^2}}}$$

Product Rule

18) y = (x)(arctan$\sqrt{(7x)}$)

$$y = (x)\arctan(7x)^{1/2}$$

$$y' = (x) \cdot \frac{1}{1+(\sqrt{7x})^2} \cdot \frac{1}{2}(7x)^{-1/2} \cdot 7 + \arctan(7x)^{1/2}$$

$$\boxed{y' = \left(\frac{x}{1+7x}\right)\left(\frac{7}{2\sqrt{7x}}\right) + \arctan\sqrt{7x}}$$

Product Rule

Find dy/dx.

19) $y = (2x)(e^x) - 2e^x$

$\frac{d}{dx}(2e^x)$

$y' = \left[2x(e^x) + e^x(2)\right] - 2e^x = 2xe^x + 2e^x - 2e^x = 2xe^x$

product rule

20) **Find dy/dx if** $y = 9^{\sin(6x)}$ (This is an exponential function with a base of 9)

$y = 9^{\sin 6x}$

$\frac{dy}{dx} = 9^{\sin 6x} \cdot \ln 9 \cdot \cos(6x) \cdot 6$

21) **Find dy/dx if** $y = \frac{\ln(5x^3)}{x^4}$ and then simplify completely

$y' = \frac{x^4 \cdot \left[\frac{1}{5x^3} \cdot 15x^2\right] - \ln(5x^3)(4x^3)}{(x^4)^2} = \boxed{\frac{3x^3 - 4x^3 \ln(5x^3)}{x^8}} = \frac{3 - 4\ln(5x^3)}{x^5}$

22) **Find dy/dx if** $y = \ln(\ln(3x))$ and then simplify completely

$\frac{1}{\ln(3x)} \cdot \frac{1}{3x} \cdot 3 = \frac{1}{x \ln(3x)}$

23) **Find dy/dx if** $y = \log_5(2x - 5x^2)$ (This is a logarithmic function with a base of 5)

$y = \log_5(2x - 5x^2)$

$y' = \frac{1}{(2x-5x^2)\ln 5} \cdot (2 - 10x) = \frac{2 - 10x}{(2x-5x^2)\ln 5}$